Beat Relationships between Orbital Periodicities in Insolation Theory

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Long-term time variations of insolation result from changes in the solar constant and the Earth's orbit around the Sun. Using modern asymptotic solutions for the Earth's orbit over the last 5 Myr, Berger (1978a) represented the eccentricity e, the precessional parameter $e \sin\tilde{\omega}$ ($\tilde{\omega}$ being the longitude of perihelion measured from the moving vernal equinox), and the obliquity ϵ in trigonometric-series expansions, from which he then tabulated the main terms. One pair of precessional-parameter terms and four pairs of eccentricity terms, however, have members that are identical except with regard to their amplitudes; since there is otherwise no physical distinction between them, their separate identities will be ignored here except for enumeration purposes.

Taking the 14 leading terms of the series expansion of e as a representative sample, we find that all of the associated periods can be exactly expressed in beat relationships, $P_{ij}^{-1} = P_i^{-1} - P_j^{-1}$, with pairs of the first six precessional-parameter periods (P_i, P_j). Therefore, they are not independent periods. Table 1 lists the 6 precessional-parameter periods and the 14 eccentricity periods ranked by decreasing amplitude of the terms, to which are added our calculated beat periods. The last are provided with the maximum number of significant figures that are warranted by Berger's published figures in each case. It appears virtually certain that all of the eccentricity periods can be derived from precessional-parameter periods. Moreover, the phases and amplitudes of Berger's published terms for the expansion of e follow predictably from those listed for the expansion of $e \sin\tilde{\omega}$. Thus, the phases for e are simply the differences of the corresponding pairs of phases for $e \sin\tilde{\omega}$; the amplitudes, however, can be predicted only in a very approximate and qualitative fashion from the rankings of the precessional-parameter amplitudes.

It is an immediate consequence that the eccentricity periods themselves must have beat relationships to each other. These relationships can be easily found by comparing pairs of precessional-parameter term indices (i, j) in the last column of Table 1. For example, the second-listed and third-listed eccentricity periods are associated with $(i, j) = (1, 3)$ and $(i, j) = (2, 3)$, respectively. The resulting beat period will be the first-listed eccentricity period because it has $(i, j) = (1, 2)$.

Berger's obliquity periods are related to each other in a similar way. A check of the ten possible pair combinations of the first five obliquity periods has been made to justify this assertion. Three of the 47 published obliquity periods are identical to published eccentricity periods—1.28, 2.04, and 3.47 Myr—and the corresponding phases also are equal. The importance of these slow periods is that they have beat relationships to the first-, second-, and fourth-listed obliquity periods, which, accordingly, must be very close to each other: 0.0410, 0.0397, and 0.0405 Myr.

The slow periods themselves have an exact beat relationship to each other, and so not all three can be fundamental. Their physical reality, however, may be questioned because Berger's calculations go back only 5 Myr. Possibly these slow periods are mixtures of the five lowest harmonic multiples (¼, ⅓, ½, ⅔, ¾) of the record length, the mixing being caused by mutual beating of the harmonics. If these periods are not physically real, much of the complex period structure in Berger's results will disappear. Longer orbital time series are needed to resolve this question.

Wigley (1976) noted long ago that orbital time-series power spectra showed a split peak near the mean precessional-parameter period of 0.0210 Myr. He pointed out that the periods of the two divided peaks, 0.0231 and 0.0188 Myr, would naturally give rise to a beat period of 0.101 Myr, a value that was interestingly close to the only eccentricity period recognized at that time. He also showed that amplitude modulation of the precessional signal by an eccentricity oscillation with a 0.10-Myr period could cause the peak splitting.

Using Berger's detailed orbital data, we have now confirmed in great detail Wigley's conjecture about a mathematical relation between the precessional parameter and eccentricity periods. Berger's leading eccentricity term, however, is not the one with a period of 0.09 Myr; instead, it has a longer period of 0.41 Myr. This period (but not any longer period) shows up in the slow modulation of the amplitudes of the predicted time variations of e, $e \sin\tilde{\omega}$, and the caloric in-

TABLE 1. Orbital periodicities from Berger's series expansions compared with calculated beat periods.

Precessional parameter term (i)	Precessional parameter period (Myr)	Eccentricity period (Myr)	Beat period P_{ij} (Myr)	Precessional parameter terms (i, j)
1	0.023716	0.412885	0.4130	1, 2
2	0.022428	0.094945	0.09494	1, 3
3	0.018976	0.123297	0.1233	2, 3
4	0.019155	0.099590	0.09960	1, 4
5	0.019261	0.131248	0.1313	2, 4
6	0.023293	2.035441*	2.031	3, 4
		0.102535	0.1025	1, 5
		1.306618	1.306	1, 6
		0.136412	0.1364	2, 5
		0.603630	0.6039	2, 6
		3.466974	3.481	4, 5
		0.102384	0.1024	3, 6
		1.282495	1.282	3, 5
		0.107807	0.1078	4, 6

* Correctly cited value from Berger (1977).

solation (see the curves in Vernekar, 1972; Berger, 1978b). It can also be seen in Milankovitch's (1941) original insolation curves (his Figs. 51 and 55), although both Milankovitch and Vernekar explicitly mentioned, in connection with the eccentricity variations, only a period of 0.09 Myr. This period of 0.41 Myr is related, as we have seen, to two *very* close precessional-parameter periods.

Our present results, which link together Berger's numerous eccentricity, obliquity, and precessional-parameter periods that he had listed without interpretation, are expected to be helpful in understanding the orbital periodicities that have been claimed in time-series analyses of long-term climatic records.

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